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Note on the measurement of short lengths

XXIV.

NOTE ON THE MEASUREMENT OF SHORT LENGTHS.

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IT is often desirable in practical astronomy to determine short linear units with such a degree of accuracy that the errors in the unit may be disregarded, in comparison with the errors of the observations in which it is involved. Such instances as the determination of the errors of micrometer screws, the single divisions of large circles, the apertures of diaphragms and ring-micrometers, the intervals between micrometer threads, may be readily cited, in which tedious numerical computations and considerable observing would be avoided, if such units could be readily submitted to an investigation under the very high magnifying power of the microscope relative to an eye-piece.

In the usual method of comparing short lengths with the microscope by means of an eye-piece micrometer, we meet the difficulty that no greater distance can be measured at one operation than can be included within the two extreme lines of the micrometer in the field of view. In this case, resort must often be had to low-power objectives, in which event the micrometer may include a desired space beyond the field of a higher power; but, at the best, the microscope eye-piece micrometer fails in all cases where so long consecutive distances as 0.1 inch are to be measured. The expense of the exquisite comparators made by Repsold, Froment, Brunner Frère, and Troughton and Simms, places them beyond ordinary reach. And the current idea that exact measures must be made with the aid of arbitrary scales, whose divisions may always be assumed to be relatively the same, is apt to cause us to overlook the extreme precision now attained in the construction of short screws, and the methods of measuring adapted to the stage of the microscope.

The screw stage micrometer suggested itself as an available way of submitting short linear units to exact measurement, provided the stand

of the microscope be made of greater stability than in usual constructions, and that the screw itself be of accurate workmanship.

It is not material in such measurements that the zero of the scale should retain a fixed position for more than a few hours together. The screw is so short that it most probably is affected throughout its length by the same conditions of temperature and thickening of the oil. And with the micrometer screw we can apply the well-known principle that a bisection of a small object can be made more exactly than can the distance of that object be estimated relative to two micrometer lines contiguous; unless, indeed, the object is placed midway between two closely parallel lines, which becomes then also a case of bisection.

In order to carry out the idea, Mr. Crouch constructed for me one of his first-class microscope stands, with some modifications in it which I thought necessary to insure the solidity we find in astronomical instruments. A clamp is added to the axis on which the instrument swings, so that it may be rigid at any inclination. The "Jackson arm" contains a small clamp so that any possible play in the rack and pinion can be counteracted. In the Crouch model, this arm has a bearing of 17 cm. in length, and 16 mm. in width.

Resting upon this bearing is the cradle which carries the body of the microscope; its base is 16 mm. in width, and the chord of its upper circular surface is 19 mm. The body, which is constructed of brass tubing, 2 mm. thick, and 36 mm. interior diameter, is soldered to this cradle. The side of the cradle away from the body carries the ordinary T rail with a smooth-working rack. The pinion is provided with large heads, 5.7 cm. in diameter, and the performance is satisfactory enough to readily focus a high angle sixth upon an object, without resort to the fine adjustment which, in the Jackson model, unfortunately alters the distance between the object-glass and the reticule in the eye-piece.

The screw and pinion moving the mechanical stage are provided with large heads 3.7 cm. in diameter, for the purpose of more easily re-setting upon the first line of a series in measuring the same space with different parts of the micrometer screw to be hereafter mentioned.

The ordinary triple-threaded screw for carrying the mechanical stage being too coarse to allow of exact setting, Messrs. Buff & Berger have replaced it for me with a screw having forty-one threads to the English inch. This screw is opposed to the micrometer screw, so that the principle of repetition may be used in measures where two contiguous lines of a scale appear in the same field of view.

The fine adjustment is provided with an unusually stiff spring, to

avoid possible change when once set. The eye-pieces are provided with close-fitting collars, so that the draw tube may be removed and the eye-pieces inserted directly into the body of the microscope.

The micrometer stage was constructed by G. & S. Merz, of Munich. It is their screw micrometer stage, adapted originally to their own microscope stands. It consists essentially of a slide moving upon a base plate 75 $\times$ 77 mm., and between two ledges adjusted with sides parallel to the slide and the axis of the micrometer screw. The slide in section is symmetrical, with its upper edge 3.35 cm., its lower edge 3.90 cm., and its vertical 5.0 mm.; its upper surface has a length of 7.4 cm. The slide carries upon its upper surface another slide, which by a rack and pinion is moved at right angles to the axis of the screw. This motion is necessary in order to assure an observer that lines of the series he may be about to observe are placed at right angles to the axis of the screw.

The slide first mentioned is pulled by spiral springs with a force varying from 0.7 kil. to 2.0 kil. against the end of the screw, as the screw moves the slide from one end of its run to the other, the bearing surface being of steel. The nut through which the screw turns is fixed to the lower plate on which the slide moves. This nut may be adjusted for position, *i.e.* to render it concentric with the screw, and its friction on the screw may be altered by turning a small screw which passes through the nut on one side. This side has been cut through, so that the small screw has really the nature of a clamp screw.

The sliding plate carries a pointer indicating whole revolutions of the screw on a silvered scale fixed to the lower plate.

The screw itself is of steel, and it is cut as nearly as practicable with 75 lines to the Paris inch. It is cut over a length of 26 mm., and is 3.7 mm. in diameter at the bottom of the screw spiral. It has the ordinary pattern micrometer head 46 mm. in diameter, which is divided into 100 parts, each of which may be subdivided into 20 parts, or even to a less degree by estimation by means of a mica scale and a small magnifying lens. The nut is of red metal, and has an upper surface rectangular in shape with a breadth of 14 mm. and a length in the direction of the screw axis of 11.1 mm., thus preserving a ratio of 3 : 1 with reference to the diameter of the screw.

It might be remarked that this ratio is an old established one; but that Mr. Adam Hilger tells me he has lately constructed some small screws, in which the relation of the nut to the diameter of the screw was disregarded, but the nut was constructed $\frac{3}{4}$ the length of the screw. He spoke highly of his success with this construction.

The screw in use is slightly oiled with an unguent consisting of equal parts of beeswax and tallow, with about $\frac{1}{30}$ part of clock oil added.

To facilitate exact setting with the screw, a smoothly turned and thin wooden disc 8.5 cm. in diameter slips over the screw head, to be clasped at its opposite edges by the fingers and thumb, in turning the screw. The whole micrometer screws to a stage plate, which may be readily slipped into the grooves cut in the stage of the microscope stand ordinarily to receive the object-holders.

The results given of the measures of short standards by this apparatus would be of little interest, unless accompanied by the results of an investigation of the errors to which a single setting of the screw is liable.

A simple method of investigating at once the errors depending upon the graduation of the head of the screw, of the variation in different parts of the same revolution, and of any cumulative error in the length of one revolution at different distances from the assumed zero of the scale, is to use a single band in the manner described below of the width of the value of one revolution, consisting of as many lines ruled on glass as there are units in the denominator of the fraction expressing the value of the smallest fractional part of the head to be considered.

The first line of this band, when the whole band has been passed over, is brought successively back to the index in the eye-piece, which should be perhaps two parallel lines nearly the same distance apart as the apparent width of the line on the stage micrometer as seen in the field of view. One of the screws of the mechanical stage is used for this purpose. This band should have lines enough upon it to have two consecutive ones in the field at once with a high power objective, in order to have its errors investigated with an eye-piece micrometer, and independently of the screw. It should be borne in mind that in this case the measures should be made in the same part of the field, to avoid errors arising from the unequal distortion of the eye-piece lenses. We thus avoid the otherwise necessary examination of a long scale of lines; and it is my opinion that it is safer to make the more numerous settings required by this method, than to trust to any inexhaustive treatment of a series of many lines, such as would be necessary without a considerable expenditure of time.

In determining the mean value of one revolution, we shall derive an advantage in using the mean of the ten settings for terminal points.

If now we put

$d =$ the number of spaces in the band,

$\gamma_0 =$ the micrometer reading on the first line,

$\gamma_1 =$ " " " " second line,

$\gamma_n =$ " " " " n^{th} line,

$M_0 =$ the mean reading of the first d lines,

$M_1 =$ " " " " second d lines,

$M_n =$ " " " " $(n+1)^{th}$ d lines,

$m_p =$ the value of 1 rev. at p revolutions,

we have

$$m_p = M_{p+1} - M_p$$

when the spaces in the band are commensurate with the value of one revolution.

We have also the accumulated error, from the p^{th} to the $(p+a)^{th}$ term, —

$$E = (m - m_p) + (m - m_{p+1}) \;.\;.\;.\;. \;(m - m_{p+a}),$$

depending on the whole revolutions.

The value of the corrections to be applied depending on the irregularities of single parts of one revolution, will be of the general form: —

$$e = \frac{m_p}{d} - \left\{ \gamma_{(pd - \frac{d}{2} + 1)} - \gamma_{(pd - \frac{d}{2})} \right\}$$

In the present case we have assumed $d = 10$, and the value of the screw is investigated for each $\frac{1}{10}$ of a revolution from $0.^{r}00$ to $25.^{r}0$ of the scale.

I am indebted to the courtesy of Prof. W. A. Rogers for a band of ten lines corresponding at 75° F. to one revolution of my screw, and so equably spaced that the spaces are sensibly uniform with any powers used in the following investigation. I should readily have detected a difference so great as 0.00001 of an inch between any two spaces of the series.

In making the observations from which the following results are derived, a $\frac{1}{5}$ objective by Crouch, having an angular aperture of 100°, and adjusted for glass cover of the slide, was combined with a short-focus negative eye-piece provided with a reticule on cover glass placed in the focus of the eye-lens. The magnifying power was 1050 diameters, nearly.

For illumination, the edge of a flame of a kerosene lamp was placed in the focus of a system of condensers 4½ inches in diameter, and a beam of rays was thrown from the distance of three feet upon the concave mirror, which reflected them centrally upon the glass plate containing the band. It was found that this illumination answered the purpose; for though the lines did not show the detail visible with monochromatic light and sub-stage condenser, yet, being comparatively widely separated, they were well adapted to measurement, when lines ruled closely would have been measured with difficulty.

The following table contains the results in millimeters of the investigation relative to periodicity of the values of one revolution. M_0 being the mean of the first ten readings of the screw, and E being the sum of the residuals of $m_{0 \text{ to } 24}$ from the mean value of one revolution.

	M_0 to 24.	m_0 to 24.	$m - m_0$ to 24.	E.	E in mm.
	r.	r.			
M 0	0.54856	.9986	—.0002	—.0002	—.00007
1	1.54717	.9970	+.0014	+.0012	+.00043
2	2.54418	.9974	+.0010	+.0022	+.00079
3	3.54153	.9975	+.0009	+.0031	+.00112
4	4.53904	.9981	—.0007	+.0024	+.00087
5	5.53712	.9980	+.0004	+.0028	+.00101
6	6.53515	.9977	+.0007	+.0035	+.00126
7	7.53384	.9995	—.0011	+.0024	+.00087
8	8.53338	.9997	—.0013	+.0011	+.00040
9	9.53306	.9987	—.0003	+.0008	+.00029
10	10.53173	.9978	+.0006	+.0014	+.00051
11	11.52950	.9994	—.0010	+.0004	+.00014
12	12.52894	.9994	—.0010	—.0006	—.00021
13	13.52798	.9986	+.0002	—.0004	—.00014
14	14.52656	.9995	—.0011	—.0015	—.00054
15	15.52608	.9989	—.0005	—.0020	—.00072
16	16.52496	.9987	—.0003	—.0023	—.00083
17	17.52363	.9985	—.0001	—.0024	—.00087
18	18.52212	.9972	+.0012	—.0012	—.00043
19	19.51934	.9978	+.0006	—.0006	—.00022
20	20.51713	.9989	—.0005	—.0011	—.00040
21	21.51561	.9975	+.0009	—.0002	—.00007
22	22.51306	.9994	—.0010	—.0012	—.00043
23	23.51241	.9981	+.0003	—.0009	—.00032
M 24	24.51046				

Mean value of $m = 0.99841$ rev.

The following table contains the residuals of the separate readings from the mean value of each revolution, also expressed in millimeters: —

VALUES OF e IN PARTS OF A MILLIMETER.

Rev.	.0 to .1	.1 to .2	.2 to .3	.3 to .4	.4 to .5	.5 to .6	.6 to .7	.7 to .8	.8 to .9	.9 to 1.0
0	+.00000	+.00002	—.00003	+.00003	— 00004	+.00003	—.00002	+.00000	+.00000	—.00001
1	—.00002	+.00001	—.00008	+.00004	+.00002	+.00000	+.00004	—.00004	—.00001	+ 00003
2	—.00002	+.00004	—.00004	+.00005	+.00000	—.00001	+.00002	—.00009	+.00005	+.00001
3	+.00002	+.00000	—.00006	+.00005	+.00001	—.00001	+.00005	—.00007	+.00002	— 00001
4	+.00000	+.00001	—.00001	+.00003	+.00001	—.00004	+.00003	—.00001	—.00001	+.00001
5	+.00001	+.00002	—.00002	—.00000	+ 00003	—.00003	+.00003	—.00001	—.00003	+.00000
6	+.00005	—.00002	+.00000	—.00001	—.00000	—.00002	+.00002	—.00001	—.00001	—.00002
7	—.00002	+ 00004	—.00002	+.00001	+.00002	—.00000	+.00003	—.00003	—.00001	+.00001
8	—.00001	—.00000	—.00003	+.00003	+.00001	+.00002	—.00000	—.00000	+.00000	—.00000
9	+.00001	+.00000	—.00005	+.00004	+.00000	+.00002	+.00000	+.00000	— 00001	—.00001
10	+.00000	—.00000	—.00001	+.00001	—.00001	+.00000	+.00003	—.00005	+.00001	+.00000
11	+.00002	+.00003	—.00000	—.00003	+.00002	+.00001	—.00001	+.00001	—.00000	—.00004
12	+.00001	+.00000	—.00001	+.00003	+.00000	+.00002	+.00001	+.00001	—.00003	—.00002
13	—.00001	+.00001	+.00001	+.00001	—.00002	+.00003	—.00001	+.00002	—.00003	— 00001
14	—.00000	+.00006	—.00003	+.00001	+.00000	—.00000	+.00004	—.00004	—.00002	—.00000
15	—.00001	—.00001	—.00002	+.00002	+.00000	+.00002	+.00000	+ 00002	—.00004	+.00001
16	—.00001	+.00001	—.00001	+.00001	—.00000	+.00005	—.00002	+.00003	—.00005	—.00000
17	+.00001	+.00002	+.00001	—.00001	—.00000	—.00001	+.00003	—.00002	—.00002	+ 00001
18	+.00002	+.00003	—.00002	+.00002	—.00002	—.00001	+.00004	—.00002	—.00004	+.00002
19	+.00000	+.00003	—.00004	—.00001	+.00002	+.00001	+.00002	+.00002	—.00005	+.00000
20	+.00003	—.00001	—.00001	+.00001	—.00003	+.00003	—.00001	—.00000	+.00000	—.00001
21	+.00000	+.00002	—.00003	+.00000	+.00001	+.00003	—.00002	+.00003	—.00000	—.00004
22	+.00004	—.00003	+.00002	—.00002	—.00001	+.00001	—.00000	—.00001	—.00002	+.00001
23	—.00005	—.00001	+.00000	+.00003	+.00000	+.00003	—.00001	+.00001	—.00001	+.00001
24	—.00001	+.00000	—.00001	+.00003	—.00002	+.00003	+.00001	—.00000	—.00000	—.00004

e in no case being so great as a unit in the fourth decimal place.

I think from the above results that we are not warranted in assigning any error of eccentricity in the screw-head, or of sensible variation in value of the different parts of the single revolutions.

There is a sensible periodic error depending upon the entire number of revolutions. This periodic error is probably a function of the pressure exerted by the springs. It is not the purpose of the present paper to discuss the absolute errors of the screw, but simply to point out their probable amount at arbitrary intervals.

In this screw, as in all screws adapted to exact measurement, it is preferable in comparing two lengths to set the screw-head at the same zero for the first line in each of the two lengths; and if the measures are made in the centre of the field the distortion of the microscope lenses is insensible.

If an eye-piece micrometer is used, it is necessary that all measures be made in the same part of the field. And if that much more exact instrument (in the writer's opinion), the filar micrometer, be applied to the eye-piece of such a microscope comparator as described above, any measure within the field will be executed with the extreme of precision. The errors of the eye-piece micrometer screw are, in this case, approximately multiplied by one-tenth of the focal length of the objective. It is necessary, however, to take the same precautions as with the eye-piece micrometer, in regard to using the same part of the field. It is also better to begin with the same zero of the micrometer head in consecutive measures, and use the same part of the screw; though of course it is not so important here as in the case of the screw stage micrometer.

The preceding remarks are based upon the following considerations relating to the distance between two lines which are seen in different parts of the field of view at the same time: —

1°. A distortion of this distance may be caused by the objective, or the eye-piece, or both.

2°. The lines of an eye-piece filar micrometer may be so illuminated that the apparent distance between two lines in the field of view is not truly measured in bisecting first one and then the other.*

3°. The filar micrometer has errors of its own screw which are variable for different parts of its length, but which probably are sensibly the same for the same interval repeatedly used within a short time.

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* Professor Newcomb, in his paper on the Uranian and Neptunian systems, Washington Astr. Obs. for 1873, points out that this source of error may be remedied by using an achromatic eye-piece. It can only occur when the micrometer lines and the object measured are apparently of different colors.

To determine the value of one revolution of the screw of the stage micrometer, and more particularly the ratio existing between the various short standards available, I give the following measures: —

Date.	Temp. F.	Object Measured.	Resulting value of 1 revol'n.	Limiting Readings of Microm. Scale.	No. of Settings.	Reduced to 70° F.
1877. Sept. 28	77°	mm. on cover-glass by Froment, of Paris .	mm. 0.36072	r. 8.1 to r. 10.9	6	mm. 0.36072
,, July 24	77°.5	the same	0 36075	9.8 to 12.6	6	0.36075
,, Dec. 7	67°.8	the same	0.360685	Mean of 3.0 to 5.8 15.5 to 18.3 and 25.5 to 28.3	12	0.360684
,, Sept. 28	77°	mm. on cover-glass marked "Secretan," Paris	0.36111	8.1 to 10.9	6	0.36111
,, July 24	77.°5	the same	0.36114	9.8 to 12.6	6	0.36114
,, Dec. 7	67°.8	the same	0.361180	3.0 to 5.8 15.0 to 17.8 25.7 to 28 5	12	0.361179
,, Dec. 7	67°.2	Lines on glass plate compared with U. S. C. S. standard centimeter	0.361330	1.0 to 36.9	6	0.361329
,, Dec. 7	67°.3	Lines on glass plate compared with U. S. C. S. "Brunner Frère" standard centimeter	0.36121	1.0 to 28.8	8	0.36121
1878. Feb. 2	63°.0	Electrotype copy of the U. S. C. S. "Brunner Frère" standard centimeter	0.361125	3.0 to 30 7	12	0.361134

The 5th column contains the limits within which the screw readings were confined, and the last column is computed by assuming

the coefficient of expansion of white glass	to be	0.00 00 086	for 1° C.
,, ,, ,, untempered steel	,,	0.00 00 108	,,
and ,, ,, ,, copper	,,	0.00 00 172	,,

and applying the small differential corrections to the results in the fourth column.

I wish to thank Mr. R. W. Willson of Harvard College for his skilful aid in making the micrometer readings necessary in the first and second tables.